Meson propagators in spontaneously broken gauge theories

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Abstract

Contrary to common belief, the longitudinal vector meson and scalar meson propagators have very different forms in spontaneously broken gauge theories. We choose the Abelian-Higgs model as an example and show that the longitudinal vector meson and scalar meson propagators have double poles by an extensive use of the Ward-Takahashi identities.

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Spontaneously broken gauge theories have been with us for a long time. A well-known example is the Electroweak Standard Model[1]. Despite being studied for several decades, some of the peculiar features of spontaneously broken gauge theories have never been mentioned in the literature. In this paper, we will give a discussion of the vector and scalar meson propagators of spontaneously broken gauge theories. Propagators in quantum field theories generally have simple poles. This is indeed the case for the vector and scalar meson propagators before symmetry is spontaneously broken. Such a simple structure does not hold anymore after the symmetry is spontaneously broken. In the following, we will show this by the extensive use of the Ward-Takahashi identities. To make our discussion simple, we will use the Abelian-Higgs model as an example. We will start with the unrenormalized Lagrangian and obtain all our results in the unrenormalized form. We will give a discussion on the renormalized case and the Electroweak Standard Model at the end of the paper.

The Lagrangian for the Abelian-Higgs theory is

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) - V \ , \tag{1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \ D_\mu\phi = (\partial_\mu + igA_\mu)\phi \ ,$$

$$V = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \ , \tag{2}$$

and g, μ^2 and λ are the gauge coupling, the two-point and four-point scalar couplings respectively.

The effective Lagrangian in the α-gauge is

$$L_{eff} = L + L_{gf} + L_{ghost} \ , \tag{3}$$

where

$$L_{gf} = -\frac{1}{2\alpha}(\partial^\mu A_\mu - \alpha\Lambda\phi_2)^2 \ , \tag{4}$$

and

$$L_{ghost} = -i\partial_\mu\eta\partial^\mu\xi + i\alpha\Lambda M\eta\xi + i\alpha g\Lambda H\eta\xi \ , \tag{5}$$

with

$$\phi = \frac{v + H + i\phi_2}{\sqrt{2}} \quad \text{and} \quad M = gv \ . \tag{6}$$

The quantity v is so defined that the vacuum expectation value of H is zero, i.e.

$$< H >= 0 \ . \tag{7}$$

The variations of the fields under the BRS[2] transformations are:

$$\delta A_\mu = \partial_\mu \xi \ , \tag{8a}$$

$$\delta \phi_2 = -g\xi(v + H) \ , \tag{8b}$$

$$\delta H = g\xi\phi_2 \ , \tag{8c}$$

$$\delta i\eta = \frac{1}{\alpha}\partial^\mu A_\mu - \Lambda\phi_2 \ , \tag{8d}$$

and

$$\delta\xi = 0 \ . \tag{8e}$$

All fields and parameters are bare.

In the Abelian-Higgs model, a longitudinal A meson may propagate into either a longitudinal A or the ϕ_2 scalar meson. Thus, all of the following propagators

$$\begin{array}{ll} < 0|TA_\mu(x)A_\nu(0)|0 > \ , & < 0|TA_\mu(x)\phi_2(0)|0 > \ , \\ < 0|T\phi_2(x)A_\mu(0)|0 > \ , & < 0|T\phi_2(x)\phi_2(0)|0 > \ , \end{array} \tag{9}$$

are non-zero and together they form a 2×2 mixing matrix. We shall denote the Fourier transform of such a propagator with the symbol G and put

$$G^{AA}_{\mu\nu}(k^2) = D^T_{AA}(k^2)T_{\mu\nu} + D_{AA}(k^2)L_{\mu\nu} \ , \tag{10a}$$

$$G^{A\phi_2}_\mu(k^2) = -G^{\phi_2 A}_\mu(k^2) = \frac{k^\mu}{k} D_{A\phi_2}(k^2) \ , \tag{10b}$$

$$G^{\phi_2\phi_2}(k^2) = D_{\phi_2\phi_2}(k^2) \ , \tag{10c}$$

where

$$T_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \ , \ L = \frac{k_\mu k_\nu}{k^2} \ .$$

and $k = \sqrt{k^2}$. The scalar ϕ_2 mixes the longitudinal A but not with the transverse A, as indicated by the factor k^μ in the expression for $G^{A\phi_2}$. Thus, in the 2×2 mixing matrix under discussion, this factor k^μ will be replaced by k. For the same reason, $L_{\mu\nu}$ can be replaced by unity. The 2×2 mixing matrix is therefore equal to

$$\begin{bmatrix} D_{AA} & D_{A\phi_2} \\ D_{\phi_2 A} & D_{\phi_2\phi_2} \end{bmatrix} . \tag{11}$$

We shall express the propagators by their 1PI amplitudes.

In a field theory without mixing, let the unperturbed propagator for a particle be $i(k^2 - m^2)^{-1}$ with m^2 its unperturbed mass squared. If we call Π to be its 1PI self-energy amplitude, then this propagator is the inverse of $-i(k^2 - m^2 - \Pi)$ or, $-i(k^2 - \Gamma)$, where $\Gamma = m^2 + \Pi$. Extending this to our case, we can express the inverse of the mixing matrix as

$$i \begin{bmatrix} \frac{k^2}{\alpha} - \Gamma_{AA}(k^2) & -ik(\Lambda - \Gamma_{A\phi_2}(k^2)) \\ ik(\Lambda - \Gamma_{A\phi_2}(k^2)) & \alpha\Lambda^2 - k^2\Gamma_{\phi_2\phi_2}(k^2) \end{bmatrix} . \tag{12}$$

where the unperturbed value of the Γ functions are $\Gamma_{AA} = M^2$, $\Gamma_{A\phi_2} = M$ and $\Gamma_{\phi_2\phi_2} = 1$.

The Ward-Takahashi identities for the propagators can now be written down easily,

$$< T(\frac{1}{\alpha}\partial^\mu A_\mu(x) - \Lambda\phi_2(x))(\frac{1}{\alpha}\partial^\nu A_\nu(x) - \Lambda\phi_2(x)) >= 0 , \tag{13}$$

$$< T(\frac{1}{\alpha}\partial^\mu A_\mu(x) - \Lambda\phi_2(x))A^\nu(y) >=< Ti\eta(x)\partial^\nu\xi(y) > , \tag{14}$$

and

$$< T(\frac{1}{\alpha}\partial^\mu A_\mu(x) - \Lambda\phi_2(x))\phi_2(x) >= -g < Ti\eta(x)\xi(y)(v + H(y)) > . \tag{15}$$

Equations (13)-(15) give

$$\frac{k^2}{\alpha^2}D_{AA}(k^2) + \frac{2ik}{\alpha}\Lambda D_{A\phi_2}(k^2) + \Lambda^2 D_{\phi_2\phi_2}(k^2) = -\frac{i}{\alpha} , \tag{16}$$

$$-\frac{ik}{\alpha}D_{AA}(k^2) + \Lambda D_{A\phi_2}(k^2) = ikD_{\eta\xi}(k^2) , \tag{17}$$

and

$$-\frac{ik}{\alpha}D_{A\phi_2}(k^2) - \Lambda D_{\phi_2\phi_2}(k^2) = -gvD_{\eta\xi}(k^2) - gD_{\eta\xi}(k^2)\Gamma_{\eta(\xi H)}(k, -k) , \tag{18}$$

where $\Gamma_{\eta(\xi H)}$ is an abnormal amplitude with ξ and H joined together.

Using (16), we get

$$\Gamma_{AA}(k^2)\Gamma_{\phi_2\phi_2}(k^2) = (\Gamma_{A\phi_2}(k^2))^2 \ . \tag{19}$$

which then gives

$$D_{AA}(k^2) = i\alpha\Gamma_{AA}(k^2)\frac{\alpha\Lambda^2 - k^2\Gamma_{\phi_2\phi_2}(k^2)}{J^2(k^2)} \ , \tag{20}$$

$$D_{\phi_2\phi_2}(k^2) = i\Gamma_{AA}(k^2)\frac{k^2 - \alpha\Gamma_{AA}(k^2)}{J^2(k^2)} \ , \tag{21}$$

and

$$D_{A\phi_2}(k^2) = -D_{\phi_2 A}(k^2) = -\alpha k\Gamma_{AA}(k^2)\frac{\Lambda - \Gamma_{A\phi_2}(k^2)}{J^2(k^2)} \ , \tag{22}$$

where

$$J(k^2) = \alpha\Lambda\Gamma_{AA}(k^2) - k^2\Gamma_{A\phi_2}(k^2) \ . \tag{23}$$

From (20)-(22), we get

$$kF_A(k^2) \equiv -\frac{ik}{\alpha}D_{AA}(k^2) - \Lambda D_{\phi_2 A}(k^2) = \frac{k\Gamma_{A\phi_2}(k^2)}{J(k^2)} \ , \tag{24}$$

and

$$F_{\phi_2}(k^2) \equiv -\frac{ik}{\alpha}D_{A\phi_2}(k^2) - \Lambda D_{\phi_2\phi_2}(k^2) = i\frac{\Gamma_{AA}(k^2)}{J(k^2)} \ . \tag{25}$$

One can see that the denominator in (20)-(22) is the square of J. Since the poles of these propagators come from the zeroes of J, and since J is a linear superposition of 1PI self-energy amplitudes, which are analytic functions of k^2, the order of the poles of the propagators are always even.

It is known that propagators in quantum field theories generally have simple poles. Contrary to common belief, one can demonstrate that this is often not true when two fields mix under the auspices of the Ward-Takahashi identities. In the case studied here, the longitudinal A and the scalar meson have the same unperturbed mass to begin with. The poles for these fields are both at $k^2 = \alpha\Lambda M$. As the interactions are turned on and the propagators form a mixing matrix, the positions of the poles change but the Ward-Takahashi identities force them to remain to be the same point. Thus the two simple poles merge to form a double pole.

On the other hand, the ghost propagator does not have the above double pole structure. Let the Fourier transform of $< 0|Ti\eta(x)\xi(0)|0 >$ be denoted as

$$D_{\eta\xi}(k^2) \ . \tag{26}$$

From (17) and (24), we get

$$D_{\eta\xi}(k^2) = -\frac{i\Gamma_{A\phi_2}(k^2)}{J(k^2)} \ . \tag{27}$$

The only gauge that still bears a simple pole for the meson propagators is the Landau gauge. This is done by setting α to go to zero. In this case, the only propagator that is nonvanishing in the mixing matrix (11) is $G^{\phi_2\phi_2}$ which takes the value

$$G^{\phi_2\phi_2}(k^2) = \frac{i}{k^2\Gamma_{\phi_2\phi_2}(k^2)} \ . \tag{28}$$

The ghost propagator decouples from the theory and takes the form

$$D^{\eta\xi}(k^2) = \frac{i}{k^2} \ . \tag{29}$$

Therefore, both the propagators of (28) and (29) are of simple poles.

If one adds fermion fields to the theory, the Lagrangian given in (1) will have additional terms. When the fermions are chiral fermions, parity is not conserved. If one assumes that the expanded Lagrangian still possess the same BRS invariance and that the fermion fields are introduced in such a way that there is no anomaly, the Ward-Takahashi identities given above still holds, and thus the vector and scalar meson propagators will still have the same form.

The above discussion can be extended easily to the Electroweak Standard Model. There are now four vector mesons to begin with. The positively charged W mesons will mix with the positively charged scalar to form a 2×2 mixing matrix. Similarly for the negatively charged mesons. The other two mesons, the Z meson and the photon (or, in terms of the original $SU(2)$ and $U(1)$ gauge fields, the neutral W meson and the B meson) will mix with the scalar partner of the Higgs and form a 3×3 mixing matrix. One can carry out a similar

but somewhat complicated derivation to demonstrate that the double pole structures again appear in the vector meson and scalar meson propagators[3].

In summary, we have shown that propagators in spontaneous broken gauge theories can have different pole structures once the symmetry is broken spontaneously. This is done by an extensive use of the Ward-Takahashi identities. The fact that the propagators have double poles would make the renormalization of spontaneously broken gauge theories, such as the Electroweak Standard Model more involved in covariant α gauges, let alone the Landau gauge. A study of this has been carried out recently[3] and will be presented elsewhere.

REFERENCES

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